UNIVERSITI MALAYSIA PAHANG
AL-SULTAN ABDULLAH

# FINAL EXAMINATION

| | | |
|---|---|---|
| **COURSE** | **:** | **APPLIED STATISTICS** |
| **COURSE CODE** | **:** | **BUM2413** |
| **COURSE COORDINATOR** | **:** | **MS. NUR ZAHIRAH BINTI MD NOOR** |
| **DATE** | **:** | **24 JUNE 2024** |
| **DURATION** | **:** | **3 HOURS** |
| **SESSION/SEMESTER** | **:** | **SESSION 2023/2024 SEMESTER II** |

## INSTRUCTIONS TO CANDIDATES:

1. This examination paper consists of **SEVEN (7)** questions. Answer **ALL** questions.
2. All answers to a new question should starts on a new page.
3. All calculations and assumptions must be clearly stated, and in **FOUR (4) decimal places**.
4. Candidates are not allowed to bring any material other than those allowed by the invigilator into the examination room.

## EXAMINATION REQUIREMENTS:

1. Statistical Tables & Formulae 2.0

## APPENDIX:

1. None

**DO NOT TURN THIS PAGE UNTIL YOU ARE TOLD TO DO SO**

This examination paper consists of **TEN (10)** printed pages including the front page.

**QUESTION 1 [13 MARKS]**

A dietitian student from a local university conducted a study to compare the average and variability of weights (in grams) for two flavors of yogurt drink, Low-Fat Strawberry Kudo and Low-Fat Blueberry Kudo. Nine samples of Low-Fat Strawberry Kudo and 13 samples of Low-Fat Blueberry Kudo were randomly selected and purchased. The study, assuming normal distribution weights, used hypothesis testing to compare two population variances as shown in **Figure 1**. It merely provides insights into the weight characteristics of these Kudo varieties for dietary considerations.

F-Test Two-Sample for Variances

$\frac{\sigma_1^2}{\sigma_2^2}$

| | *Low-Fat Strawberry Kudo* | *Low-Fat Blueberry Kudo* |
|---|---|---|
| $\bar{x}$ Mean | 21.0333 | 20.8923 |
| $s^2$ Variance | 0.3675 | 1.0141 |
| $n$ Observations | 9 | 13 |
| df | 8 | 12 |
| F | 0.3624 | |
| P(F<=f) one-tail | 0.0787 ×2 | |
| F Critical one-tail | 0.3045 | |

**Figure 1**

i) Based on **Figure 1**, can we conclude that there is a difference in population variance of the weights of both Low-Fat Strawberry Kudo and Low-Fat Blueberry Kudo? (Use the *P*-value approach).

ii) Based on the results from **i)**, conduct appropriate hypothesis testing to determine whether the population mean weights of Low-Fat Strawberry Kudo is not similar to Low-Fat Blueberry Kudo.

**[13 Marks]**

**[CO2, PO3, C4]**

i) $H_0 : \sigma_1^2 = \sigma_2^2$

$H_1 : \sigma_1^2 \neq \sigma_2^2$ (claim)

P-value $= 0.1574 > \alpha = 0.05$, do not reject $H_0$.

At $\alpha = 0.05$, we have insufficient evidence to support the claim.

ii) based on i), equal variances.

$H_0 : \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 - \mu_2 \neq 0$ (claim)

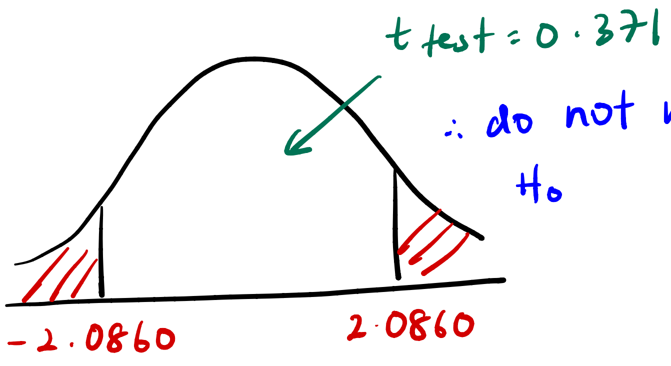
$$s_p = \sqrt{\frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{8(0.3675) + 12(1.0141)}{9 + 13 - 2}}$$

$$= 0.8692$$

$$t_{test} = \frac{(\bar{x}_1 - \bar{x}_2) - \mu_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{\alpha,\nu} = \frac{(21.0333 - 20.8923) - 0}{0.8692 \sqrt{\frac{1}{9} + \frac{1}{13}}}$$

$$= 0.3741$$

$t_{0.025,\,20} = 2.0860$




$\therefore$ do not reject $H_0$

At $\alpha = 0.05$, we have insufficient evidence to support the claim.

**QUESTION 2 [8 MARKS]**

A quality controller at Everedy, a battery company, is evaluating the variability in the lengths of their batteries. To comply with company regulations, the variance of battery length should ideally be 0.4. In the study, a random sample of 20 AAA batteries was selected for analysis.

$\alpha = 1 - 0.90 = 0.10$

i) Based on the sample, the 90% confidence interval for the standard deviation of battery length lies between 0.3394 mm and 0.5859 mm. Using the confidence interval approach, conduct a hypothesis test to determine if the variance complies with the company's regulation.

ii) Identify the type of error (Type I, Type II or no error) exists in **i)**. Justify your answer.

**[8 Marks]**

**[CO2, PO3, C4]**

i) CI for sd : (0.3394, 0.5859)

CI for variance : (0.1152, 0.3433)

$H_0 : \sigma^2 = 0.4$ (claim)

$H_1 : \sigma^2 \neq 0.4$

Since 0.4 is NOT within (0.1152, 0.3433), reject $H_0$.

At $\alpha = 0.1$, we have insufficient evidence to support the claim.

ii) Type I

**QUESTION 3 [20 MARKS]**

Agricultural researchers are conducting an experiment to determine the effects fertilizer type and watering frequency on the yield of a certain crop. They tested on three types of fertilizers (A, B, and C) and three watering frequencies (daily, every three days, and weekly). They want to analyze if there are any significant differences in crop yield (in kilograms) based on these two factors. The output of analysis from *Microsoft Excel* at 2% level of significance is given in **Figure 2**.

D E W

ANOVA

| *Source of Variation* | *SS* | *df* | *MS* | *F* | *P-value* | *F crit* |
|---|---|---|---|---|---|---|
| Sample | 0.9630 | 2 | 0.4815 | 0.1121 | 0.8946 | 4.9001 |
| Columns | 442.7407 | 2 | 221.3704 | 51.5259 | 0.0000 | 4.9001 |
| Interaction | 13.9259 | 4 | 3.4815 | 0.8103 | 0.5348 | 3.8369 |
| Within | 77.3333 | 18 | 4.2963 | | | |
| | | | | | | |
| Total | 534.9630 | 26 | | | | |

**Figure 2**

i) State the independent and dependent variables. dependent: crop yield

independent: types of fertilizers and watering frequency

ii) How many treatments are involved in this experiment? List all possible treatments.

$3 \times 3 = 9$,

A, D B, D C, D
A, E B, E C, E
A, W B, W C, W

iii) Test at 2% level of significance whether there is an interaction effect between type of fertilizer and watering frequency using the *P*-value approach.

iv) Based on your answer in **iii)**, conduct the row effect and column effect tests.

**[20 Marks]**

**[CO2, PO3, C4]**

$H_0$ : There is No interaction between type of fertilizer and watering frequency.

$H_1$ : There is AN interaction between type of fertilizer and watering frequency.

P-value = 0.5348 > $\alpha$ = 0.02, do not reject $H_0$.

At $\alpha$ = 0.02, there is no interaction between type of fertilizer and watering frequency.

## ROW EFFECT ← mentioned first

$H_0$: There is no effect of type of fertilizer

$H_1$: There is an effect of type of fertilizer

P-value = 0.8946 > $\alpha$ = 0.02, do not reject $H_0$.

At $\alpha$ = 0.02, there is no effect of type of fertilizer

## COLUMN EFFECT

$H_0$: There is no effect of watering frequency

$H_1$: There is an effect of watering frequency

P-value = 0.0000 < $\alpha$ = 0.02, reject $H_0$.

At $\alpha$ = 0.02, there is an effect of watering frequency.

**QUESTION 4 [16 MARKS]**

Heat treating is a typical method employed in carburizing metal parts such as gears, where the thickness of the carburized layer is crucial for reliability. To ensure the quality, a lab test is conducted on each furnace load. The test is destructive, involving cross-sectioning an actual part and soaking it in a chemical for a specified period. The aim of this lab test is to investigate the dependency of soak time (in seconds) on the pitch (in the module). **Figure 3** displays the scatter diagram, while **Figure 4** presents the *Microsoft Excel* output derived from the sample data collected during the lab test.

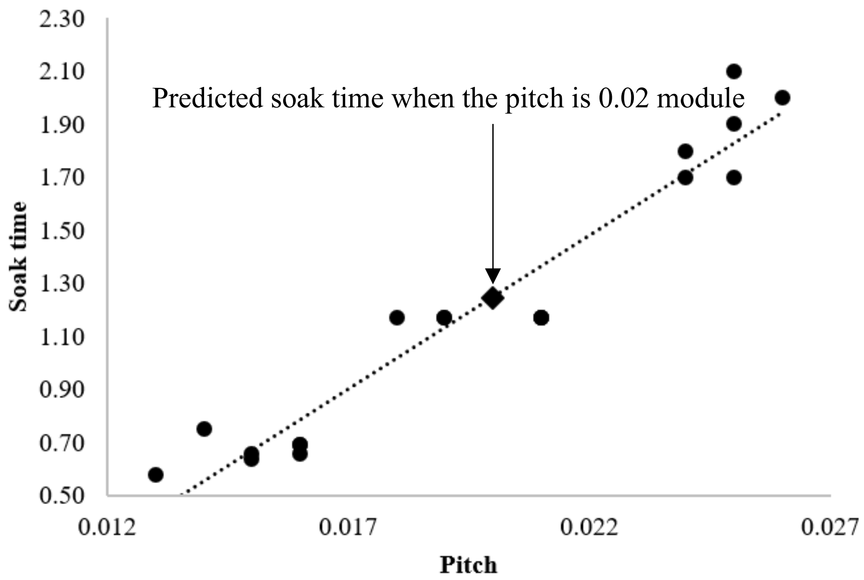



**Figure 3**

SUMMARY OUTPUT

| Regression Statistics | |
|---|---|
| Multiple R | 0.9640 |
| R Square | 0.9293 |
| Adjusted R Square | 0.9252 |
| Standard Error | 0.1414 |
| Observations | 19 |

ANOVA

| | df | SS | MS | F | Significance F |
|---|---|---|---|---|---|
| Regression | 1 | 4.4690 | 4.4690 | 223.5018 | 0.0000 |
| Residual | 17 | 0.3399 | 0.0200 | | |
| Total | 18 | 4.8089 | | | |

P-value for linearity test

| | Coefficients | Standard Error | t Stat | P-value | Lower 95% | Upper 95% | Lower 95.0% | Upper 95.0% |
|---|---|---|---|---|---|---|---|---|
| Intercept | -1.0715 | 0.1557 | -6.8829 | 0.0000 | -1.3999 | -0.7430 | -1.3999 | -0.7430 |
| Pitch | 115.9468 | 7.7557 | 14.9500 | 0.0000 | 99.5838 | 132.3098 | 99.5838 | 132.3098 |

$\beta_1$ +ve → r +ve

**Figure 4**

i) Interpret the correlation coefficient between the soak time and the pitch.

r = 0.9640. There is a strong, positive linear relationship between soak time and pitch.

ii) Briefly describe whether the answer in **i)** can be supported based on the scatter diagram in **Figure 3**.

Yes since scatter plot also shows a positive linear trend.

iii) Test the linearity between the soak time and the pitch by using the *P*-value approach.

next page

iv) Identify and interpret the coefficient of determination between the soak time and the pitch.

$r^2 = 0.9293$. 92.93% of the variation in the soak time can be explained by pitch

v) Predict the soak time when the pitch is 0.02 module by using an appropriate estimated simple linear regression line.

$y = -1.0715 + 115.9468(0.02)$

$= 1.2474$ seconds.

vi) Based on **Figure 3**, do you think the predicted soak time in **v)** is reasonable? State your reason.

Yes. The predicted value visually located on the estimated SLR line.

**[16 Marks]**

**[CO2, PO3, C4]**

iii) $H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

p-value = 0.0000 < $\alpha$ = 0.05, reject $H_0$.

At $\alpha = 0.05$, there is a significant linear relationship between the soak time and the pitch.

**QUESTION 5 [18 MARKS]**

An engineer performed an experiment to determine an effect of carbon dioxide ($CO_2$) temperature (in °C), peanut moisture (in %) and peanut particle size (in mm) on the total yield of peanut oil. An experiment was conducted on 20 batches of peanut and a multiple linear regression analysis was performed. The following *Microsoft Excel* output shown in **Figure 5** is a multiple linear regression analysis done by the engineer.

SUMMARY OUTPUT

| *Regression Statistics* | |
|---|---|
| Multiple R | 0.8569 |
| R Square | 0.7343 |
| Adjusted R Square | 0.6845 |
| Standard Error | 14.1754 |
| Observations | 20 |

ANOVA

| | *df* | *SS* | *MS* | *F* | *Significance F* |
|---|---|---|---|---|---|
| Regression | 3 | 8887.4643 | 2962.5 | 14.7429 | 0.0001 |
| Residual | 16 | 3215.0857 | 200.94 | | |
| Total | 19 | 12102.55 | | | |

| | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 90.0%* | *Upper 90.0%* |
|---|---|---|---|---|---|---|---|---|
| Intercept | 65.5829 | 9.3086 | 7.0454 | 0.0000 | 45.8495 | 85.3163 | 49.3312 | 81.8347 |
| CO2 temperature | 0.3577 | 0.0937 | 3.8153 | 0.0015 | 0.1589 | 0.5564 | 0.1940 | 0.5213 |
| peanut moisture | 0.6536 | 0.6562 | 0.9960 | 0.3341 | -0.7375 | 2.0446 | -0.4921 | 1.7992 |
| peanut size | -14.1568 | 2.3689 | -5.9760 | 0.0000 | -19.1787 | -9.1349 | -18.2927 | -10.0209 |

**Figure 5**

i) What is the dependent variable in this regression analysis?

total yield of peanut oil

ii) Determine the number of independent variables involved in this analysis.

3

iii) State the correlation coefficient and interpret its value.

$r = 0.8569$ : There is a strong linear relationship between CO2 temperature, peanut moisture, peanut size and total yield of peanut oil.

iv) State the regression coefficient of peanut size ($\hat{\beta}_3$) and interpret its value.

$\beta_3 = -14.1568$. Holding other variables constant, when the peanut size increase by 1mm, the total yield of peanut oil decrease by 14.1568.

v) Test the hypothesis that at least one of the independent variables has a significant relationship with the dependent variable at 10% significance level. Use the *P*-value method.

next page

vi) The summary of the multiple linear regression analysis is given in **Table 1**. Complete the table for the three predictors based on **Figure 5**.

**Table 1**

| Predictor | *P*-value | R Square | Adjusted R Square | Regression Equation |
|---|---|---|---|---|
| $x_1$ | 0.0923 | 0.1414 | 0.0937 | $\hat{y} = 39.9929 + 0.2643x_1$ |
| $x_2$ | 0.7527 | 0.0057 | -0.0496 | $\hat{y} = 52.15 + 0.37x_2$ |
| $x_3$ | 0.0010 | 0.4472 | 0.4165 | $\hat{y} = 87.5029 - 11.877x_3$ |
| $x_1, x_2$ | 0.2737 | 0.1414 | 0.0404 | $\hat{y} = 39.9929 + 0.2643x_1 + 0.0000x_2$ |
| $x_1, x_3$ | 0.0000 | 0.7179 | 0.6847 | $\hat{y} = 70.1370 + 0.3732x_1 - 13.7635x_3$ |
| $x_2, x_3$ | 0.0031 | 0.4927 | 0.4330 | $\hat{y} = 78.8550 + 1.0708x_2 - 12.6504x_3$ |
| $x_1, x_2, x_3$ | 0.0001 | 0.7343 | 0.6845 | $\hat{y} = 65.5829 + 0.3577x_1 + 0.6536x_2 - 14.1568x_3$ |

vii) Based on your answer in **vi)**, select the best regression model for studying the yield of peanut oil. Justify your answer for the selection.

Best model is the one with $x_1, x_3$ variables. Lowest P-value and highest adjusted $R^2$.

viii) Based on your answer in **vii)**, predict the yield of peanut oil if the batch of peanut used has a size of 3mm with 17% moisture and set at 75°C $CO_2$ temperature.

**[18 Marks]**

**[CO2, PO3, C4]**

$$y = 70.1370 + 0.3732(75) - 13.7635(3)$$
$$= 56.8365$$

v) $H_0: \beta_1 = \beta_2 = \beta_3 = 0$

$H_1$: At least one of the independent variables is related to the dependent variable.

p-value $= 0.0001 < \alpha = 0.1$, reject $H_0$.

At $\alpha = 0.1$, at least one of the independent variables is related to the dependent variable.

**QUESTION 6 [12 MARKS]**

A Bluetooth speaker manufacturing company has a defective rate of 7%. The number of defective speakers produced by the company in a week was observed for a year (52 weeks). The relative frequencies and probability for defective speakers are recorded in **Table 2**.

**Table 2**

| **Number of defective speakers** | **0** | **1** | **2** | **3** | **4** | |
|---|---|---|---|---|---|---|
| **Frequency** | 26 | *m* | 7 | 3 | 5 | 52 |
| **Probability** | 0.0081 | 0.0756 | 0.2646 | 0.4116 | 0.2401 | |
| Expected | 0.4212 | 3.9312 | 13.7592 | 21.4032 | 12.4852 | |

i) Determine the value of *m* in **Table 2**.

$52-(26+7+3+5) = 11$

ii) Can we conclude that the number of defective speakers produced by the company in a week follows the Binomial distribution with parameter $n = 4$ and $p = 0.07$ at 5% significance level?

**[12 Marks]**

**[CO2, PO3, C4]**

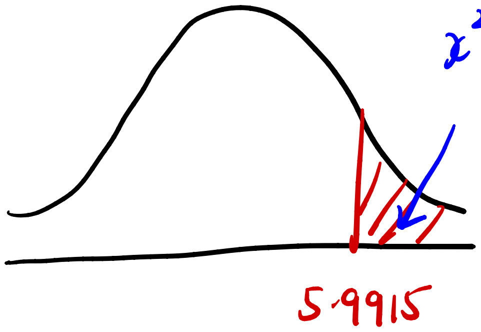
$H_0$: The number of defective speakers produced by the company in a week follows the Binomial distribution with parameter $n=4$ and $p=0.07$

$H_1$: The number of defective speakers produced by the company in a week does not follow the Binomial distribution with parameter $n=4$ and $p=0.07$

$$x^2_{test} = \frac{(44-18.1116)^2}{18.1116} + \frac{(3-21.4032)^2}{21.4032} + \frac{(5-12.4852)^2}{12.4852}$$

$$= 57.3157$$

$\chi^2_{0.05, 2} = 5.9915$




$\chi^2$ test $= 57.3157$

reject $H_0$.

At $\alpha = 0.05$, the number of defective speakers produced by the company in a week does not follow the Binomial distribution with parameter $n = 4$ and $p = 0.07$.

**QUESTION 7 [13 MARKS]**

A survey was conducted on holiday destinations preferences between man and woman. A random sample of 165 man and 185 woman participated in the survey resulted the information shown in **Table 3**.

**Table 3**

| Gender | Holiday destinations | | | | |
|---|---|---|---|---|---|
| | Islands and Beaches | Forests and Mountain | Town and City | Theme Park | |
| Man | 49.0286 34 | 39.1286 $p$ | 32.0571 26 | 44.7857 50 | 165 |
| Woman | 54.9714 70 | 43.8714 28 | 35.9429 42 | 50.2143 45 | 185 |
| | 104 | 83 | 68 | 95 | 350 |

i) Identify one of the variables involved in this survey.

gender / holiday destinations

ii) Determine the value of $p$ in **Table 3**.

55

iii) Test at 2.5% significance level that the holiday destinations preference is dependent of gender.

**[13 Marks]**
**[CO2, PO3, C4]**

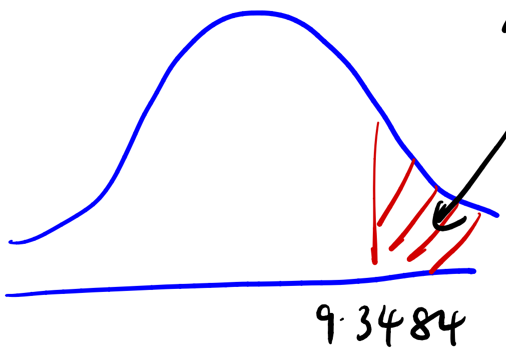
$H_0$ : Holiday destinations preference and gender is independent.

$H_1$ : Holiday destinations preference and gender is dependent.

$$\chi^2 \text{ test} = \frac{(34-49.0286)^2}{49.0286} + \frac{(70-54.9714)^2}{54.9714} + \frac{(55-39.1286)^2}{39.1286} + \frac{(28-43.8714)^2}{43.8714} + \frac{(26-32.0571)^2}{32.0571} + \frac{(42-35.9429)^2}{35.9429} + \frac{(50-44.7857)^2}{44.7857} + \frac{(45-50.2143)^2}{50.2143} = 24.2087$$

**END OF QUESTION PAPER**

$\chi^2_{0.025,3} = 9.3484$




$\chi^2_{test} = 24.2087$

reject $H_0$

At $\alpha = 0.025$, holiday destinations preference and gender is dependent.